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ABSTRACT

The application of the programmable calculator to evaluating complicated formulas is illustrated by considering the formula for finding the area of any triangle when only the lengths of the three sides are known. Other advantages of the programmable calculator are discussed such as freeing the student to explore more challenging problems and discovering unexpected results. Flow charts and programs for the problem stated are given. (MP)

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"Using Programmable Calculators to Evaluate Complicated Formulas"

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Learning to evaluate and apply formulas is an important problem solving skill. Unfortunately, many formulas are given little, if any, consideration in the school curriculum because the computations required are tedious and time consuming. Since programmable calculators perform calculations easily and quickly, these powerful machines can convert the drudgery of evaluating formulas into simple programming exercises. Furthermore, by letting a programmable calculator do most of the tedious work, students become free to explore more challenging problems and discuss more fruitful mathematical issues.

To illustrate this valuable application of programmable calculators, consider the problem of finding the area of any triangle when only the lengths of the three sides are known. The desired area number is given by Heron's Formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where a, b, and c, are the lengths of the three sides and s is the semiperimeter,

$$s = \frac{a + b + c}{2}$$

The formula is easily derived by one method that merits special mention. It is taken from an excellent geometry text (Hoffer, 1979, p. 402) and gives carefully thought out, easy to follow steps for guiding students in the derivation of the formula. Even though the message of this article is not the derivation of Heron's Formula, Hoffer's Method is presented for the convenience of interested readers who may find it helpful in their teaching.

- (i) Apply the Pythagorean Theorem to triangles I and II in figure 1.
- (ii) Solve for x in terms of a, b, and c.
- (iii) Express h in terms of a, b, and c.

(iv) Express the area of triangle ABC in terms of a, b, and c.

(v) Let $s = \frac{a+b+c}{2}$, one half the perimeter of $\triangle ABC$, and show that

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Except for carefully chosen values of a, b, and c, the computation required by Heron's formula involves tedious calculations. Consequently, this general area formula is used infrequently. This need not be the case if a programmable calculator is available.

Figures 2 and 3 present flow charts and programs for computing the area of a triangle with Heron's formula. The program in figure 2 is for the Texas Instruments TI 57 programmable calculator and the program in figure 3 is for the Hewlett Packard HP 33 E programmable calculator. Each program can be rerun by merely repeating the initialization sequences given. These machines have been chosen because they are the least expensive fully programmable models of the two largest domestic manufactures. Since these machines are both now quite inexpensive, they seem to be the calculators most likely to have the greatest impact on the secondary school curriculum.

Using programmable calculators to evaluate complicated formulas offers advantages other than just making calculations less time consuming. By letting the calculator do most of the tedious work, the student is often free to explore more challenging problems. Again, using Heron's formula as an example, students with access to programmable calculators might be invited to consider questions such as these:

(1) For what integer values of a, b, and c, is the area of a triangle a positive integer?

(2) For isosceles triangles, Heron's formula becomes

$$A = \sqrt{s(s-a)(s-b)^2}$$

Are there values of a and b for which the area, A,

is a positive integer?

(3) For equilateral triangles, Heron's formula becomes

$$A = \sqrt{s(s-a)^3}$$

Are these integers values of "a" for which the area,

A, is a positive integer?

By gathering data for the above problems students not only gain additional insight into the nature and use of Heron's formula, but they may be motivated to form conjectures and develop proofs. For example, they will find no integer value of "a" which gives an integer value for the area of an equilateral triangle. Does this mean there is no such integer? This leads quite naturally to the formation of a conjecture and development of a proof. To show that no integer exists giving an integer value of A, substitute $\frac{3a}{2}$

for S in

$$A = \sqrt{s(s-a)^3}$$

to obtain

$$A = \frac{1}{4}a^2\sqrt{3}$$

and analyze the nature of the product of $\frac{1}{4}a^2$ and the $\sqrt{3}$.

Experimenting with programmed formulas can also lead to surprisingly beneficial, unexpected results. This is illustrated with an additional question that also can be tackled using Heron's formula.

(4) Take a length of string and tie the two ends together so that a loop of length 24 cm is formed. Next stretch the string taut into a triangle. What triangle formed in this way has the largest area?

Readers will recognize that the answer is an equilateral triangle. However, students exploring the question by checking various values of a , b , and c with the calculator may obtain unexpected but meaningful results. For example, side lengths of 4, 6 and 14 which total the required perimeter of 24 give an error message on the calculator. Actually the error message occurs because it is algebraically impossible to take the square root of a negative number (and obtain a real number result). However, there is a more important geometrical interpretation of this error message. It indicates that a triangle of side lengths 4, 6, and 14 cannot be formed. This fact, in turn, can lead to a discussion of the triangle inequality.

It is indeed unfortunate that in today's curriculum applying complicated formulas, like Heron's area formula is rarely done because of the required time that would be taken away from an already tight syllabus to do the tedious calculations. Instead, programmable calculators should be introduced to overcome this realistic time limitation. Consequently, students would have a much easier and more successful time applying the formulas to meaningful problems. Discussions of important mathematical concepts, like irrational numbers and the triangle inequality, could then take place more naturally and sooner than in the present curriculum.

While this article discusses just one formula and some of the implications of using programmable calculators to evaluate it, the authors contend that many other complicated formulas can be similarly used in the school curriculum. Readers are encouraged to search for such formulas, try them in their classrooms and share them with colleagues.

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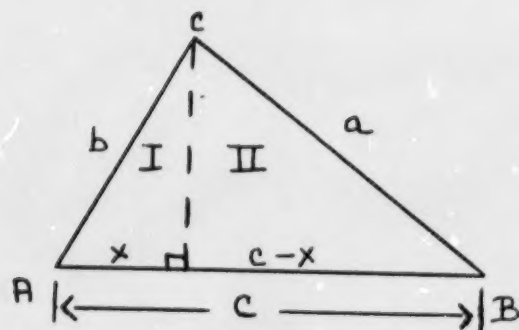
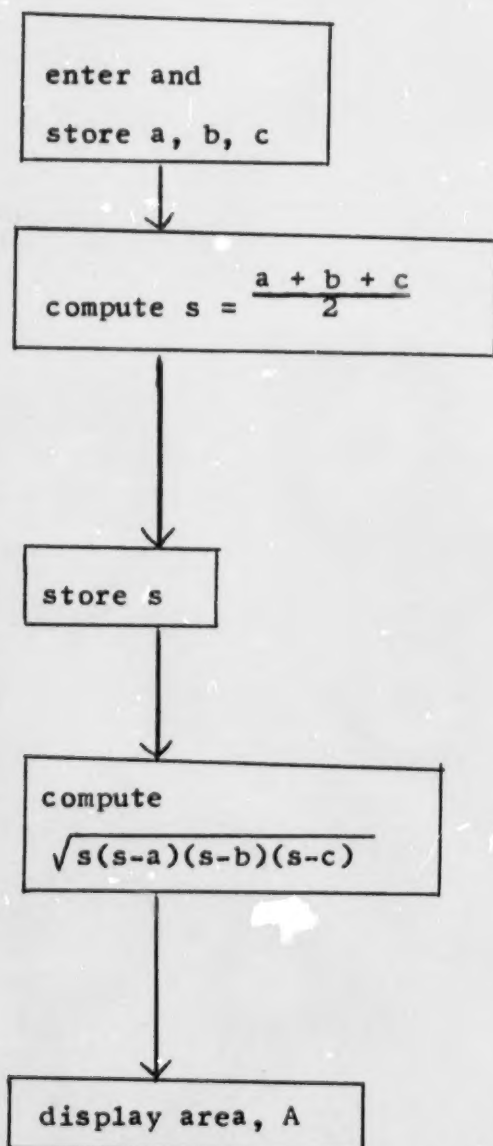


Figure 1

Flow Chart



Program

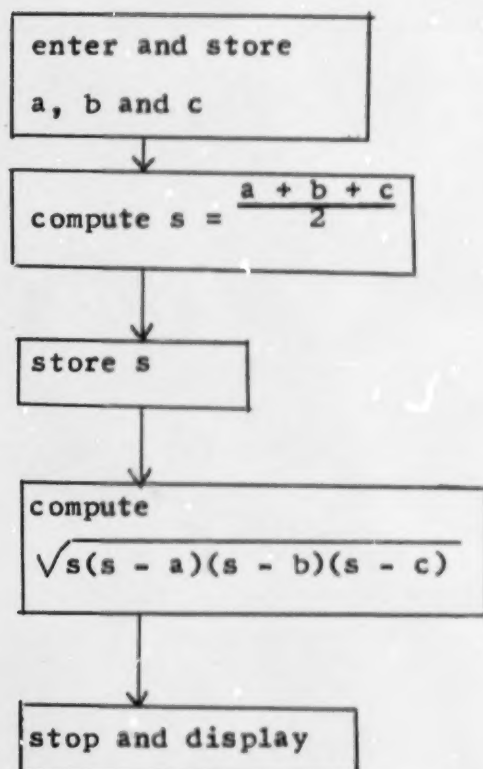
00 STO 1	03 R/S	
01 R/S	04 STO 3	
02 STO 2		
05 RCL 1	10 =	
06 +	11 ÷	
07 RCL 2	12 2	
08 +	13 =	
09 RCL 3		
14 STO 4		
15 RCL 4	22 x	29 (
16 x	23 (	30 RCL 4
17 (	24 RCL 4	31 -
18 RCL 4	25 -	32 RCL 3
19 -	26 RCL 2	33)
20 RCL 1	27)	34 =
21)	28 x	35 √x
36 R/S		

memory usage: $R_1 = a$, $R_2 = b$, $R_3 = c$, $R_4 = s$

initialize: CLR, RST, enter a, R/S enter b, R/S, enter c, R/S

Figure 2

Flow Chart



Program

```

[00 (R/S)  02 R/S  04 R/S
01 STO 7  03 STO 6  05 STO 5

06 +      08 2
07 +      09 ÷

10 STO 4
11 RCL 4  16 RCL 6  20 RCL 5
12 RCL 7  17 -      21 -
13 -      18 x      22 x
14 x      19 RCL 4  23 f sqrt x
15 RCL 4

24 GTO 00
  
```

Memory Usage: $R_7 = a$, $R_6 = b$, $R_5 = c$, $R_4 = s$

Initialize: f PRGM, enter a, R/S, enter b, R/S, enter c, R/S

Figure 3